Finite-Element Analysis of Large Spur and Helical Gear Systems

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The application of the three-dimensional finite-element substructure method has helped to improve the accuracy of calculation of gear tooth contact and fillet stresses in large spur and helical gear systems. The development of a dedicated preprocessor has reduced significantly the manual effort involved in the analysis. The methodology is explained using the example analyses of the gear systems in the two stages of a typical speed reduction gearbox used in the Pratt & Whitney-Canada PW100 turbopropeller engines.

Nomenclature

A_1,A_2,A_3 = constants in the expression for the line load intensity
B_1,B_2,B_3,B_4 = constants in the expression for normal displacement
L = length of line of contact on the tooth contact face
n = number of nodes along a contact line
$p(x)$ = line load intensity as a function of distance
R_i = reactive force at node i
x = distance along line of contact on the tooth contact face
δ_i = normal displacement at node i
$\delta(x)$ = normal displacement distribution as a function of x

Introduction

THE examination of the structural integrity of large speed reduction gear systems requires a proper assessment of the tooth contact and fillet stresses. In calculating these stresses, it is important to consider gears as complete structures rather than as pairs of teeth in mesh, as emphasized in Ref. 1 through a two-dimensional analysis of an epicyclic gear stage. In large offset speed reduction gear systems, the problem is, in addition, three-dimensional in nature. The tooth contact load distribution is influenced significantly by the stiffness of the foundations—the rim, web, and shaft. The formulation of the finite-element method using the substructure concept is suited ideally to include all aspects of the static problem. Reference 2 describes an application of this method for an assemblage of gear teeth in mesh. Easy generation of the required finite-element network is important to study the effects of different gear parameters in a cost-effective manner. Such a study can be used to attempt weight reduction.

An overview of the advanced gearbox technology as applied to the Pratt & Whitney-Canada PW100 series of turbopropeller engines is presented in Ref. 3. This paper details the features and application of the in-house analysis and network generation programs for offset spur and helical gear trains used in the above-mentioned class of engines.

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Static Stress Analysis Procedure

Tooth Contact Stresses

The transfer of torque from the pinion (driver) to the gear (driven) takes place along a differing number of tooth pairs. This number changes as the gears roll in and out of mesh. At any instant, it depends on whether it is a high- or low-contact-ratio spur or helical gear pair. In the "just-loaded" state, the line of contact along the tooth face can be calculated using involute geometry relations. In the loaded state, the contact is over surfaces around these "lines" of contact. The variation of the contact load intensity along these surfaces depends on the distribution of the combined stiffness in the normal direction of the gear and pinion. This stiffness is a result of several parameters related to the teeth—pressure and helix angle, chordal thickness, radius of contact, etc.—and to the foundation—rim, web, shaft, etc.

The finite-element method with three-dimensional representation of the pinion and gear structures can internally compute the stiffness variations. If nodes are identified on the pinion, and gear tooth contact faces and displacement boundary conditions in the normal direction are given, the contact stresses can be calculated by the analysis program. However, there are two problems:

1) Surface of contact is over a narrow area around the line of contact; this area changes with applied torque, making the problem nonlinear.

2) As is evident from Hertz formulas for bearing stresses between nonconformal cylinders, the contact area around the line of contact is very "narrow," requiring very refined mesh density.

The method that is found practical is to assume the contact to be along the "line" derived from involute geometry. Displacement boundary conditions are given along the direction of tangency of the pinion and gear-base circles at the nodes identified along these "lines." The reactive forces at these nodes from the finite-element analysis can be converted to distributed line load intensities. The contact stresses then can be computed using standard Hertz formulas for spur or helical gears as given in Ref. 4. This procedure does not take into account the effect of the Hertzian contact deformation on the distributed line load intensity. For flexible gear systems used in aerospace applications, this effect is not considered significant.

The conversion of consistent nodal reactive forces R_i at the contact nodes ($i = 1, n$) to distributed line load intensities $p(x)$ can be performed for any one tooth pair using the following procedure:

1) Knowing the normal displacements of the contacting nodes δ_i, obtain a cubic regression fit of the equation for this displacement $\delta(x)$ as a function of the distance x along the

"line" of contact as

$$\delta(x) = B_1x^3 + B_2x^2 + B_3x + B_4$$

2) Assume that the distributed line load intensity $p(x)$ can be expressed as a parabolic variation with the distance x as

$$p(x) = A_1x^2 + A_2x + A_3$$

3) Obtain the constants in the preceding equation using Force equilibrium:

$$\sum_{i=1}^{n} R_i = \int_0^L p(x)\,\mathrm{d}x$$

Work equilibrium:

$$\sum_{i=1}^{n} R_i\delta_i = \int_0^L p(x)\delta(x)\,\mathrm{d}x$$

Moment equilibrium:

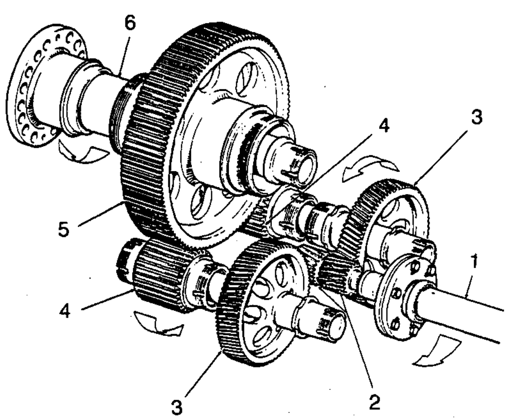
$$\sum_{i=1}^{n} R_ix_i = \int_0^L p(x)x\,\mathrm{d}x$$

Tooth Fillet Stresses

The calculation of the fillet stresses is also not straightforward. It is difficult to include a refined fillet representation in the global model of the pinion and gear teeth and their foundations without excessively enlarging the number of nodes. It is found more practical to perform the analysis in two steps. In the first step, the tooth load intensities are found with a network that represents the fillet area approximately. At this step, the displacements at the boundary surrounding the tooth fillets but "sufficiently" away also are stored. These displacements are imposed in the second step as boundary conditions on the refined network local to the fillets. The procedure is explained further through example analyses in a later section.

Analysis Program Features

The finite-element analysis software uses isoparametric solid elements with curved quadrilateral or triangular cross section. The program has many attractive features, such as the acceptance of boundary conditions along user-defined coordinate axes and the substructure concept with restart capability. The restart feature is found very useful in conducting parametric studies economically. When some parameters are changed and large portions of the network remain unaffected, most of the stiffness matrices are not recalculated. For example, the effect of misalignment can be examined by modifying the contact node boundary conditions (to include a gap between some nodes), and the stiffness matrices for the substructures need not be recalculated.



1. TORQUE SHAFT ASSEMBLY
2. 1ST STAGE HELICAL PINION
3. HELICAL GEARS
4. 2ND STAGE SPUR PINIONS
5. OUTPUT GEAR
6. PROPELLER SHAFT

Fig. 1 PW100 speed reduction gearbox.

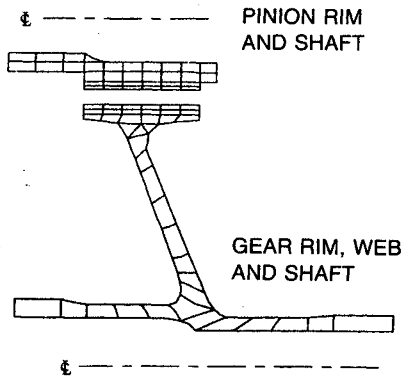


Fig. 2 Helical gear pair input two-dimensional cross section.

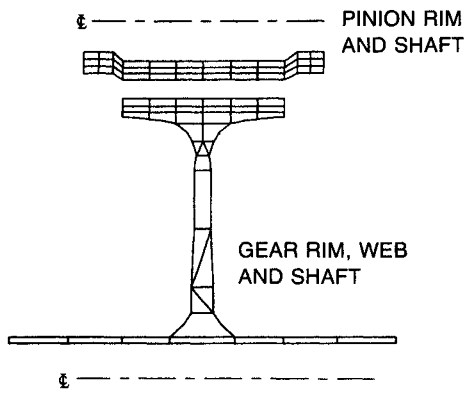


Fig. 3 Spur gear pair input two-dimensional cross section.

Automatic Network Generation Features

As discussed earlier, the static analysis can be used to explore weight reduction potentials by studying the effect of changes in parameters (such as the rim and web cross sections, pressure, and helix angles, etc.) on the stresses. The proper computation and application of torque input loads, bearing boundary conditions on the shaft, connection between the substructure boundary nodes, etc., are cumbersome and error-prone. The identification of the nodes on the lines of contact of the teeth pair in the "just-loaded" state using involute geometry is tedious, especially for helical gears. The transfer of boundary displacements form the first to the second step in the analysis for fillet stresses, explained earlier, is laborious. All of these are automated using a preprocessing software. The required inputs are the load and gear data—e.g. pitch diameter, chordal thickness, etc.—and the two-dimensional definition of the cross section of the gear and pinion rim, web, and shaft. Significant productivity improvement benefits through reduction in manual effort and elapsed time can be realized.

Example Analyses

Example analyses of a spur and a helical gear system are presented to explain the methodology. The gear train resembles the Pratt & Whitney PW100 turbopropeller engine speed reduction gearbox as illustrated in Fig. 1. The power turbine

Table 1 Gear data

	Stage 1 reduction (Helical)		Stage 2 reduction (HCR[a] spur)	
	Pinion	Gear	Pinion	Gear
Speed, rpm	19,980	4,945	4,945	1,200
No. of teeth	25	101	33	136
Diametral pitch, per in.	12.5	12.5	9.5	9.5
Pressure angle, deg	25.0	25.0	17.0	17.0
Helix angle, deg	20.2843	20.2843	—	—
Arc tooth thickness, in.	0.1299	0.0949	0.1655	0.1482
Fillet radius, in.	0.0140	0.0210	0.0590	0.0477
Axial facewidth, in.	1.70	1.70	3.00	3.00

[a]High contact ratio.

drives the first-stage double-helical (herringbone) pinion and power is split between two layshaft gears. Each layshaft drives a second-stage pinion via flexible spline couplings. Both second-stage pinions then combine their power to drive a single bull gear, which then directly drives the propeller shaft.

Basic gear geometry data are given in Table 1. Such data and a two-dimensional cross section of the pinion and gear tooth foundation (rim, shaft, etc.) form the input to the network for the fillet stress-concentration study is shown in Fig. 6. This three-dimensional network is generated automatically using the gear geometry data partially given in Table 1. the first- and second-stage gear pairs are shown in Figs. 4 and 5. These networks are generated by rotation of the two-dimensional cross sections (shown in Figs. 2 and 3) and by adding the three-dimensional networks of the meshing gear teeth. The models have been simplified in the examples by excluding the web lightening holes. The network generation software can model the actual web through another option. In this case, the input should additionally include a two-dimensional network of the web surface. A typical refined gear network for the fillet stress-concetration study is shown in Fig. 6. This three-dimensional network is generated automatically using the gear geometry data partially given in Table 1.

The analysis is carried out for a power level of 2000 shp and a propeller speed of 1200 rpm. The input torque loads are applied uniformly at the pinion bore. The torque reaction is simulated through tangential restraints at the gear bore. In the first stage, only one helical pinion is modeled and the symmetry boundary conditions are imposed on one end of the pinion shaft. The contact nodes on the pinion and gear teeth are forced to have equal displacement in the contact direction, as explained earlier. For most cases, these conditions will result in compressive contact reactions at these nodes. If tensile reactions do develop, especially for misaligned gears, an iterative contact analysis should be performed, as explained briefly in the next section.

Figures 7 and 8 show the results of the gear pair finite-element analysis. The gear tooth load distributions were obtained by mathematically transforming the discrete nodal reactions to continuous line loads by satisfying force, energy, and moment equivalences. Figure 9 illustrates the helical gear teeth load distributions across the face width with inclined lines of contact typical of helical teeth. Maximum contact stresses based on the Hertzian formulas also are shown. These stresses are primarily dependent on the material stiffness, contact load intensity, and the radii of curvature of the contacting surfaces of gear teeth and are important in establishing the pitting endurance.

The fillet stresses are calculated by imposing boundary displacements obtained by the analysis of the full network shown in Figs. 4 and 5 to the refined network shown in Fig. 6. The definition of the boundary displacements for the refined network is imposed automatically by the preprocessor software. The distribution of the gear teeth fillet stresses is shown in Fig. 10 for the middle gear tooth of the first and second stages. The stresses are plotted for the tooth fillet that is in tension, as indicated in Fig. 6. The loading on this tooth

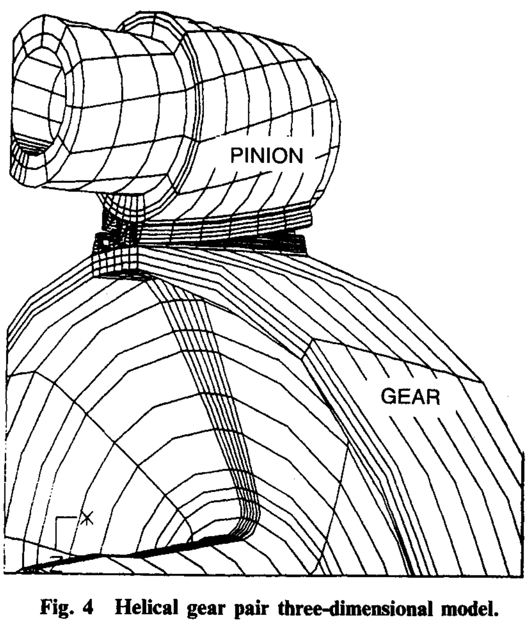


Fig. 4 Helical gear pair three-dimensional model.

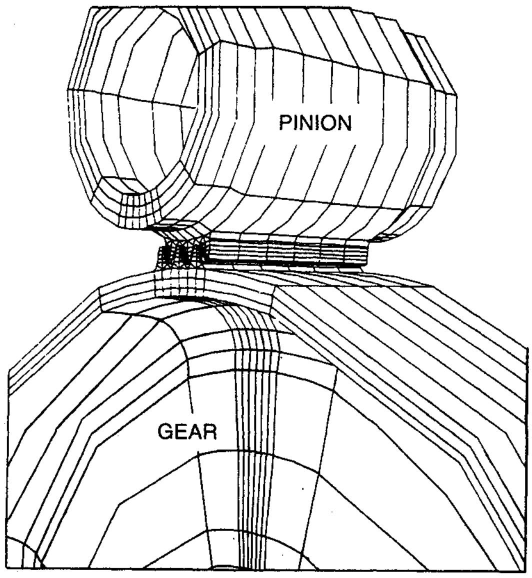


Fig. 5 Spur gear pair three-dimensional model.

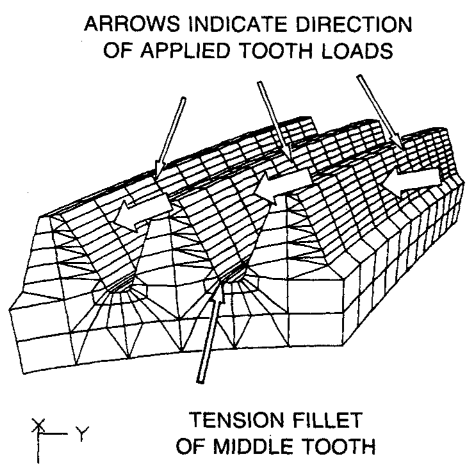


Fig. 6 Three-dimensional model for fillet stress.

Table 2 Gear tooth loads and stresses

	Stage 1 reduction (Helical)			Stage 2 reduction (HCR[b] spur)	
	Lead	Middle	Trailing	Lead	Middle
Tooth load, lb	834	1,557	930	2,705	4,199
Load share, %	25	47	28	39	61
Load distribution factor[a]	2.613	2.246	1.220	1.398	1.256
Maximum contact Stress, ksi	207	233	191	108	150
Maximum gear fillet stress, ksi	—	96	—	—	33

[a]Load distribution factor = (Peak/average) tooth load for the gear pair. [b]High contact ratio.

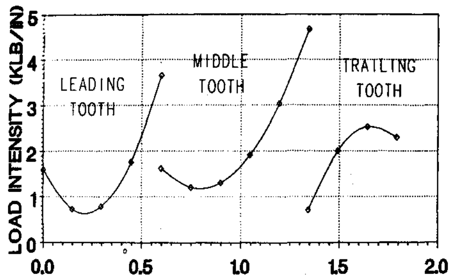


Fig. 7 Helical gear tooth load distribution.

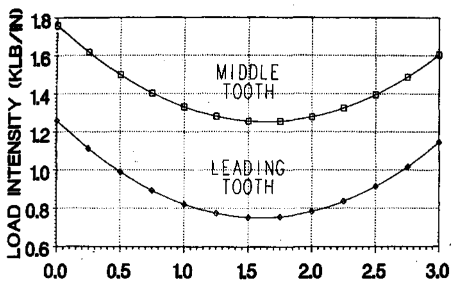


Fig. 8 Spur gear tooth load distribution.

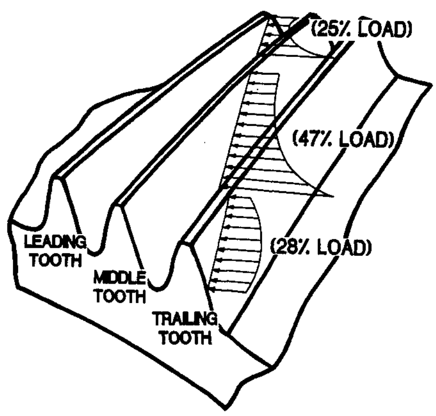


Fig. 9 Helical gear teeth loads.

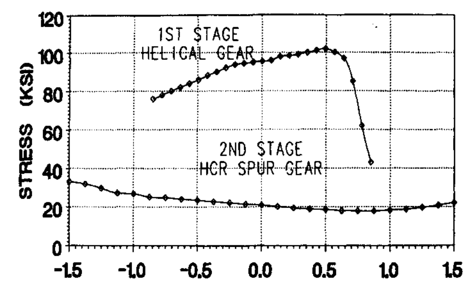


Fig. 10 Middle gear tooth fillet stress.

is along the pitch line for the second stage with spur gears, whereas it is along an inclined line passing through the pitch point for the first stage with helical gears. The load sharing among the contacting pairs of teeth, the nonuniform load distribution in each tooth, and the maximum contact and fillet stresses are summarized in Table 2.

Possible Analysis Extensions

Several extensions to the analyses described in the previous section are possible. The examination of the effect of tooth profile modifications—crowning or misalignment due to lead error, bearing bore tolerances, etc.—can be examined by changing the contact boundary conditions to include a "gap" between the tooth nodes. Recalculation of the structure stiffness can be avoided using the restart feature. When the misalignment angle or crown drop is excessively high, some tooth nodes with initial "gap" may not come in contact even at full loads. In this case, the three-dimensional model is used to find only the stiffness matrix of "probable" contact nodes in the contact "plane." The nonlinear contact problem is solved using an iterative two-dimensional program in this contact plane. Refer to Ref. 3 for additional information. The study of different instances of loading and moving lines of contact is possible since the network can be rotated and new nodes of contact of gear and pinion teeth can be identified and connected. The network generation software already is extended to cover the three-dimensional analysis of the sun/planet epicyclic gear arrangement.

Conclusions

The application of the substructure finite-element analysis and the use of dedicated preprocessor programs have helped to examine large gear systems in detail without requiring excessive manual effort. Weight reduction options can be examined more confidently through such analyses, with resultant reduction in costly experimental means. A database of calculated stresses on similar gear systems with development running experience at different power levels can be used to set allowable limits.

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